Research Guru
Online Journal of Multidisciplinary Subjects (ISSN : 2349-266X)
Volume-13, Issue-1, June 2019
Impact Factor: 4.081
www.researchguru.net

CONSTRUCTION OF ORTHONORMAL BASIS USING FRACTIONAL DIFFERENTIATION

LAL CHANDRA PATEL
Department Of Mathematics
Neharu gram bharati (deemed to be university) jamunipur kotawa dubawal Allahabad,
Email: lal9026153385@gmail.com

Abstract:
In general we know that there are many basis of $L^2(R)$ – space , in which some can be written in the terms of sinx and cosx .another way of producing an orthonormal basis from single function involves translations and modulation .but in this paper we construct an orthomormal basis of $L^2(R)$ from given basis with help of fractional differentiation which is different from parent basis .

Key words : inner product , Fourier transformation , orthonormal basis fractional differentiation.

Introduction :

Fractional differentiation is generalization of differentiation and integration of any (non integeral) order.Concept of fractional differentiation is came in existence about three hundred years ago .frist question arise by Leibnitz in 1695 . latter many mathematician work on it as Liouville, Riemann, Weyl, Fourier, Abel, Lacroix, .now differential formula for any order of cosx is given as

$\frac{d^{\upsilon}}{x^{\upsilon}}\cos(x) = \cos(x + \upsilon\frac{\pi}{2})$ where v is any number 1

There are many basis of $L^2(R)$ some of them are given by , Let $g = \chi_{[0,1]}$ and $g_{m,n}(x) = e^{2\pi imx}g(x-n)$ for m,n ε Z . we can see that $\{g_{m,n} : m , n \epsilon Z\}$ is an othonormal basis for $L^2(R)$ .There more other example of orthonormal basisgiven below as

(i) $\left\{\sqrt{2}\text{sine}\left(\frac{2k+1}{2}\pi x\right); k = 0,1,2,3,...\right\}$

(ii) $\{\sqrt{2}\text{sine}(k\pi x)\}; k = 1,2,3, ...$

(iii) $\left\{\sqrt{2}\cos\left(\frac{2k+1}{2}\pi x\right)\right\}, k = 0,1,2,3 .....$ 2

(iv) $\{1, \sqrt{2}\cos(k\pi x)\}, k = 1,2,3,4, ...$

Each one of the system is an orthonormal basis of $L^2([0,1])$.

Preliminaries:

The Fourier transformation of a function f ε $L^1(R) \cap L^2(R)$ is define by

$\hat{f}(\xi) = \int_{-\infty}^{+\infty} f(x)e^{-\imath\xi x}dx$ 3

Inner product of two function define as

$< f, g > = \int f\bar{g}$, 4

Definition ;

We say that two function f and g are orthogonal when inner product is zero .

$< f g > = 0$.

A sequence of function $\{f_n\}, n\varepsilon Z$ is an orthonormal sequence if ,

$<f_m, f_n> = \delta_{m,n}$ where,

$$\delta_{m,n} = \begin{cases} 1\ , if\ n\ = m \\ 0\ , if\ n\ \neq m \end{cases} \qquad 5$$

BALIAN – LOW THEOREM :

Suppose g ε $L^2(R)$ and $g_{m,n}(x) = e^{2\pi imx} g(x-n),\ m,n\ \varepsilon\ Z$

If $\{g_{m,n} : m\ ,n\ \epsilon\ Z\}$ is orthonormal basis for $L^2(R)$, Then either

$\int_{-\infty}^{+\infty} x^2 |g(x)|^2\, dx = \infty\ \ or\ \ \int_{-\infty}^{+\infty} \xi^2\, |\widehat{g(x)}|^2 d\xi = \infty$.

DISCUSSION: Take a sequence of function from 2 (iii) ,

$\left\{\cos\left(\frac{2k+1}{2}\pi x\right),\ k = 0\ ,1\ ,2 \ldots\ldots\ldots..\right\}$ in $L^2[0\ ,1]$.

Then $g_k(x)$ = $\cos(\frac{2k+1}{2}\pi x)$ so g(x) = $g_0(x)$ = g(∏x/2)

If $\frac{d^\nu}{dx^\nu}\cos\left(\frac{\pi x}{2}\right) = \cos(\frac{\pi x}{2} + \nu\frac{\pi}{2}) = g^\nu(\frac{\pi x}{2})$

Now Fourier transformation of g(x) WHICH GIVE tha

$$\widehat{g^\nu}(\xi) = \int_{-\infty}^{+\infty} g^\nu(x) e^{-i\xi x} dx$$

$$= \int_0^1 \cos(\frac{\pi x}{2} + \nu\frac{\pi}{2}) e^{-i\xi x} dx$$

$$= \int_0^1 \frac{1}{2}\left(e^{i\nu\frac{\pi}{2}} e^{i(\frac{\pi}{2}x - \xi)} + e^{-i\frac{\pi}{2}} e^{-i(\frac{\pi x}{2} + \xi)}\right)$$

$$= \frac{1}{i(\pi - 2\xi)}\left[e^{i(\frac{\pi}{2} - \xi)} - 1\right] e^{i\nu\frac{\pi}{2}} + \frac{1}{-i(\pi+2\xi)}\left[e^{-i(\frac{\pi}{2}} + \xi) - 1\right] e^{-i\nu\frac{\pi}{2}}$$

$$= \frac{1}{i(\pi - 2\,\xi)}[\sin\xi + i\cos\xi - 1] e^{i\nu\frac{\pi}{2}} - \frac{1}{i(\pi + 2\xi)}[-\sin\xi - i\cos\xi - 1] e^{-i\nu\frac{\pi}{2}}$$

$$= \frac{2\pi e^{i(\frac{\nu\pi}{2} - \xi)}}{(\pi^2 - 4\xi^2)} + \frac{2\cos\left(\frac{\nu+1}{2}\right)\pi + 4i\sin(\frac{\nu+1}{2})\pi}{(\pi^2 - 4\xi^2)}$$

Now we find that $|\,\hat{g}^\nu(\xi)\,|^2\,\xi^2 = \dashrightarrow 0$,hence

Now $\int_{-\infty}^{+\infty} |\hat{g}^\nu(\frac{\pi x}{2})|^2 \xi^2\, d\xi$ = finite.

Next we check that

$\int_{-\infty}^{+\infty} |g(x)|^2 x^2 dx = 2\int_0^\infty g(x)|^2 x^2\, dx = \lim_{c \dashrightarrow \infty} \int_0^c |g(x)|2x^2 =$

$\lim_c \int_0^c |\cos(\frac{\pi x}{2} + \nu\frac{\pi}{2})|^2\, x^2\, dx = \dashrightarrow \infty$

Hence general differentiation of any orthonormal basis is form another orthonormal basis.

CONCLUSION:

THE ABOVE DISCUSSION SHOW THAT THE GENERAL DIFFERENTIATION OF AN BASIS FORM ANOTHER ORTHONARMAL BASIS OF $L^2(R)$ OR $L^2[0\ ,1]$.WE CAN ALSO DERIVED THE RESULT TO OTHER BASIS WHICH ARE GIVEN ABOVE .

References:

1. A mathematical model for simulation of a water table profile between two parallel subsurface drains using

fractional derivatives

Behrouz Mehdinejadiani a, Abd Ali Naseri b, Hossein Jafari c, Afshin Ghanbarzadehd,

Dumitru Baleanu e

journal homepage: www.elsevier.com/locate/camwa

2.Fractional model for malaria transmission under control strategies

Carla M.A. Pinto a,∗, J.A. Tenreiro Machadob

3. WHO expert committee on malaria. WHO technical report series, vol. 892, Twentieth report, 2000.

4. J.C. Beier, G.C. Müller, W. Gu, K.L. Arheart, Y. Schlein, Attractive toxic sugar bait (ATSB) methods decimate populations of Anopheles malaria vectors
in arid environments regardless of the local availability of favoured sugar-source blossoms, Malaria Journal 11 (31) (2012).

5.T.K. Berthoud, M.N. Manaca, D. Quelhas, R. Aguilar, C. Guinovart, L. Puyol, A. Barbosa, P.L. Alonso, C. Dobano, Comparison of commercial kits to measure
cytokine responses to Plasmodium falciparum by multiplex microsphere suspension array technology, Malaria Journal 10 (1) (2011) 115.

6. M.E. Patarroyo, R. Amador, P. Clavijo, A. Moreno, F. Guzman, P. Romero, R. Tascon, A. Franco, L.A. Murillo, G. Ponton, G. Trujillo, A synthetic vaccine
protects humans against challenge with asexual blood stages of plasmodium-falciparum malaria, Nature 332 (1988) 158–161.

7. P.M. Graves, H. Gelband, Vaccines for preventing malaria (SPf66), Cochrane Database of Systematic Reviews 2006 (2) (2009) Art. No.: CD005966.

8 . E. Malkin, F. Dubovsky, M. Moree, Progress towards the development of malaria vaccines, Trends Parasitol. 22 (7) (2006) 292–295.

9. M.E. Halloran, C.J. Struchiner, Modeling transmission dynamics of stage-specific malaria vaccines, Parasitology Today 8 (3) (1992) 77–85.